**Assignment 5**

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**Part I:**

**(a)**

For an arbitrary series, ρ1 = PACF(1) = 0.54302.

**(b)**

Because PACF is significant until at lag = 2, here we write Yule-Walker equation for AR(2) process.

(1.2)

(1.1)

For AR(2) process, Φ2 = PACF(2) = **-0.48256**.

Substitute ρ1 = 0.54302, ρ2 = 0.04540, Φ2 = -0.48256 into (1.1), then

Therefore, Φ1 =0.80506, and Φ2 = -0.48256.

**(c)** Estimate the mean of the stationary AR(2) process:

Therefore,

= 62.62427

Since, , and C=62.62427, the estimated model for Y is:

**(d)** Estimate the autocorrelation at lag 4 for Y series

Solution:

For AR(2) model:

**(e)** under the condition that analyst incorrectly models the Y series using an AR(4), we estimate the coefficient Φ4 by using mathematical deduction that coefficient of the p-th order term equals the partial autocorrelation at lag = p in model, which means Φ4= PACF (4)= -0.05285. By observing the partial autocorrelation table, we obtain that lag 4 is within the standard error boundaries, and conclude that there is not significant evidence to suggest that 𝜙4 is different than zero. Therefore, Φ4 is statically insignificant.

**Part II (a):**

| **Parameter Estimates** | | | | | |
| --- | --- | --- | --- | --- | --- |
| **Variable** | **DF** | **Parameter Estimate** | **Standard Error** | **t Value** | **Pr > |t|** |
| **Intercept** | 1 | 0.90414 | 0.34084 | 2.65 | 0.0086 |
| **UNEMP** | 1 | -0.06468 | 0.03853 | -1.68 | 0.0948 |
| **INFL** | 1 | -72.93310 | 8.18607 | -8.91 | <.0001 |
| **PARTY** | 1 | -0.22991 | 0.14402 | -1.60 | 0.1120 |

**Hypothesis testing for unemployment rate (UNEMP)**

H0: β1= 0; the unemployment rate has non-influence on the earning index

Ha: β1 < 0; the unemployment rate has a negative influence on the earning index

According to the parameter estimates table above, we obtain that the p-value of UNEMP 0.0948 is bigger than 0.05, and conclude that we retain the null hypothesis at 95% confidence level; therefore, there is non-significant evidence to prove that unemployment rate has an effect on the earning index.

**Hypothesis testing for Inflation rate (INFL)**

H0: β2 = 0; the inflation rate has non-influence on the earning index

Ha: β2 < 0; the inflation rate has a negative influence on the earning index

According to the parameter estimates table above, we reject the null hypothesis at 95% confidence level based on P-value is smaller than 0.0001, and conclude that there is significant evidence to prove that inflation rate has a negative effect on the earing index

**Hypothesis testing for political party in power (PARTY)**

H0: β3 = 0, the political party has non-influence on the earning index

Ha: β3 < 0, the political party has a negative influence on the earning index.

According to the parameter estimates table above , we retain the null hypothesis at 95% confidence level based on P-value: 0.1121> 0.05, and conclude that there is non-significant evidence to prove that political party has an effect on the earning index

**Part II (b):**

| **Autocorrelation Check for White Noise** | | | | | | | | | |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **To Lag** | **Chi-Square** | **DF** | **Pr > ChiSq** | **Autocorrelations** | | | | | |
| **6** | 49.84 | 6 | <.0001 | -0.446 | 0.178 | -0.030 | 0.047 | -0.066 | 0.076 |
| **12** | 53.08 | 12 | <.0001 | 0.024 | -0.059 | 0.009 | 0.057 | -0.011 | 0.088 |
| **18** | 56.48 | 18 | <.0001 | -0.023 | 0.113 | -0.020 | 0.011 | 0.001 | 0.042 |
| **24** | 69.31 | 24 | <.0001 | 0.007 | -0.026 | 0.140 | -0.150 | 0.116 | 0.007 |

Hypothesis testing for white noise:

H0: Residual series {εt} is unautocorrelated to the indicated lag, that is ρ1= ρ2 = ρ3=…=0.

Ha: At least one lag is differ than 0

By observing the table above, we rejected the null hypothesis at 95% confidence level based on p-value of lag 6, lag 12, lag 18, and lag 24, all of those, are smaller than 0.0001, and conclude that at least one autocorrelation from lag 0 to lag 24 is differ than zero; therefore, the residual terms are not white- noise.

**Part II (C )**

| **Partial Autocorrelations** | | |
| --- | --- | --- |
| **Lag** | **Correlation** | -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1 |
| **1** | -0.44616 | |           \*\*\*\*\*\*\*\*\*|  .                 | |
| **2** | -0.02687 | |                 . \*|  .                 | |
| **3** | 0.04915 | |                 .  |\* .                 | |
| **4** | 0.06963 | |                 .  |\* .                 | |
| **5** | -0.03509 | |                 . \*|  .                 | |
| **6** | 0.02977 | |                 .  |\* .                 | |
| **7** | 0.09292 | |                 .  |\*\*.                 | |
| **8** | -0.02193 | |                 .  |  .                 | |
| **9** | -0.05337 | |                 . \*|  .                 | |
| **10** | 0.04822 | |                 .  |\* .                 | |
| **11** | 0.05895 | |                 .  |\* .                 | |
| **12** | 0.12579 | |                 .  |\*\*\*                 | |

| **Autocorrelations** | | | | |
| --- | --- | --- | --- | --- |
| **Lag** | **Covariance** | **Correlation** | -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1 | **Std Error** |
| **0** | 0.257353 | 1.00000 | |                    |\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*| | 0 |
| **1** | -0.114820 | -.44616 | |           \*\*\*\*\*\*\*\*\*|  .                 | | 0.070535 |
| **2** | 0.045690 | 0.17754 | |                 .  |\*\*\*\*                | | 0.083401 |
| **3** | -0.0077238 | -.03001 | |                 . \*|  .                 | | 0.085261 |
| **4** | 0.011994 | 0.04661 | |                 .  |\* .                 | | 0.085313 |
| **5** | -0.016960 | -.06590 | |                 . \*|  .                 | | 0.085440 |
| **6** | 0.019679 | 0.07647 | |                 .  |\*\*.                 | | 0.085693 |
| **7** | 0.0061471 | 0.02389 | |                 .  |  .                 | | 0.086031 |
| **8** | -0.015174 | -.05896 | |                 . \*|  .                 | | 0.086064 |
| **9** | 0.0023006 | 0.00894 | |                 .  |  .                 | | 0.086265 |
| **10** | 0.014541 | 0.05650 | |                 .  |\* .                 | | 0.086270 |
| **11** | -0.0028274 | -.01099 | |                 .  |  .                 | | 0.086454 |
| **12** | 0.022589 | 0.08777 | |                 .  |\*\*.                 | | 0.086461 |

According to the autocorrelation table above, we observe that autocorrelations of lag 0 and lag 1 are statistically significant, and the autocorrelation function decays quickly to 0 after lag 2; therefore, we can conclude that the residual series is autocorrelated and stationary. Hence, we will need to determine the order of the autoregressive process by observing the partial autocorrelation table above, which demonstrate that only lag 1 is statistically significant, and lag 2 is not statistically significant. Consequently, we decided to use AR(1) to model the residual series.

**Part II (d).** By incorporating an AR(1) process––that is, a first-order autoregressive process––into the original model, we obtained a corrected model as shown below:

where at is the white-noise term.

Table 4.1. Parameter estimates

| **Variable** | **DF** | **Estimate** | **Standard Error** | **t Value** | **Approx Pr > |t|** |
| --- | --- | --- | --- | --- | --- |
| **Intercept** | 1 | 0.8970 | 0.2162 | 4.15 | <.0001 |
| **UNEMP** | 1 | -0.0650 | 0.0242 | -2.69 | 0.0079 |
| **INFL** | 1 | -69.4850 | 6.5029 | -10.69 | <.0001 |
| **PARTY** | 1 | -0.2270 | 0.0917 | -2.47 | 0.0142 |
| **AR1** | 1 | 0.4484 | 0.0640 | 7.01 | <.0001 |

Since P-values for coefficients in Table 4.1 are all smaller than α = 0.05, all coefficients in this model are significantly different than zero on a statistical basis.

Table 4.2 Model parameters of multivariate regression in (a).

| **Ordinary Least Squares Estimates** | | | |
| --- | --- | --- | --- |
| **SSE** | 51.7279627 | **DFE** | 197 |
| **MSE** | 0.26258 | **Root MSE** | 0.51242 |
| **SBC** | 318.807922 | **AIC** | 305.594702 |
| **MAE** | 0.30575741 | **AICC** | 305.798784 |
| **MAPE** | 878.713674 | **HQC** | 310.941344 |
| **Durbin-Watson** | 2.8921 | **Total R-Square** | 0.2956 |

Table 4.3 Model parameters of corrected regression

| **Unconditional Least Squares Estimates** | | | |
| --- | --- | --- | --- |
| **SSE** | 41.3653534 | **DFE** | 196 |
| **MSE** | 0.21105 | **Root MSE** | 0.45940 |
| **SBC** | 279.4012 | **AIC** | 262.884676 |
| **MAE** | 0.30751106 | **AICC** | 263.192368 |
| **MAPE** | 1005.07363 | **HQC** | 269.567978 |
| **Durbin-Watson** | 2.0269 | **Transformed Regression R-Square** | 0.3858 |
|  |  | **Total R-Square** | 0.4367 |

According to Table 4.2 and 4.3, MSE (mean squared error) of the corrected model (0.21105) is smaller than the original model (0.26258), which suggests the autoregressive model has a higher explanatory power. A higher total R-square (0.4367>0.2956) also indicates the same fact.

**Part II (e).** Table 5.1 White-noise test for residuals

| **Autocorrelation Check for White Noise** | | | | | | | | | |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **To Lag** | **Chi-Square** | **DF** | **Pr > ChiSq** | **Autocorrelations** | | | | | |
| **6** | 3.13 | 6 | 0.7923 | -0.013 | -0.005 | 0.073 | 0.005 | -0.039 | 0.090 |
| **12** | 9.46 | 12 | 0.6629 | 0.038 | -0.076 | 0.004 | 0.075 | 0.062 | 0.113 |
| **18** | 16.46 | 18 | 0.5603 | 0.075 | 0.143 | 0.035 | -0.001 | 0.024 | 0.064 |
| **24** | 25.02 | 24 | 0.4049 | 0.010 | 0.035 | 0.105 | -0.088 | 0.095 | 0.092 |

Because the residual term series {at} is constructed zero-mean and homoscedastic, χ2 testing, which tests for autocorrelation, sufficiently tests for whether the time series {at} is a white-noise series. According to Table 5.1, all P-values are greater than the significance level of 0.05; therefore, we cannot reject null hypotheses that autocorrelation of {at} at all lags are statistically equal to zero. Consequently, we retain null hypotheses that autocorrelation at all lags are equal to zero, and conclude that the residual term at is statistically a white-noise term.

**Part II (f).** The corrected model can be expresses as follows:

where at is the white-noise term.

According to Table 4.1, all P-values for the three coefficients of IV are smaller than the 0.05 significance level. For each of these tests, null hypothesis is H0: βi = 0, and alternative hypothesis is Ha: βi ≠ 0, where βi stands for the i-th variable (UNEMP, INFL, and PARTY). As a result, we reject all of those three null hypotheses at 95% confidence level, and conclude that each of coefficient is significantly different than zero on a statistical basis; therefore, all three variables are statistically significant and have negative directional influence due to the negative coefficient value of -0.065, -69.458, and -0.227 respectively.